MODERN ANALYSIS AND GEOMETRY 2019

A CONFERENCE TO CELEBRATE GAVEN MARTIN'S 60th BIRTHDAY

JANUARY 24–25 2019, MASSEY UNIVERSITY, ALBANY

ABSTRACTS OF TALKS

1. **Kari Astala (Helsinki, Finland)**

   Title: Random tilings and non-linear Beltrami equations

   Abstract: Over the years I have had the privilege of working with Gaven Martin on a great variety questions, often related to Beltrami equations and quasiconformal structures in one way or the other.

   In this talk I will explain how Beltrami equations can be used in describing scaling limits of random tilings. This time the work is joint with Erik Duse, Istvan Prause and Xiao Zhong.

2. **Martin Bridson (Oxford, UK)**

   Title: Kähler groups and subdirect products of surfaces

   Abstract: There is much that we do not know about the fundamental groups of smooth complex projective varieties and compact Kahler manifolds. After a brief survey, I'll describe recent works of Llosa Isenrich, myself and others that provide new examples of Kähler groups, on the one hand, and new obstructions to being Kähler, on the other hand. The subgroups of direct products of surface groups play a prominent role in the discussion.

3. **Marston Conder (Auckland, NZ)**

   Title: Group actions on pseudo-real surfaces

   Abstract: A Riemann surface is called <u>pseudo-real</u> if it admits anti-conformal (orientation-reversing) automorphisms, but no anti-conformal automorphism of order 2, or equivalently, if the surface is reflexible but not definable over the reals. Such surfaces correspond to the points in

the moduli space of compact Riemann surfaces of given genus with real moduli.

In this talk I'll describe a number of discoveries about pseudo-real surfaces made over the last 10 years, including the facts that there exists one of genus g for every $g > 1$, and that the upper bound of $12(g-1)$ on the maximum number of automorphisms is attained for infinitely many $g > 1$. More recent work (with Emilio Bujalance and Javier Cirre) has produced pseudo-real analogues of theorems of Wiman and Maclachlan respectively on the maximum order of a cyclic or abelian group of automorphisms of the surface.

Finally, I'll report on some joint work with Stephen Lo (PhD student) on the pseudo-real genus $\psi(G)$ and the strong pseudo-real genus $\psi^*(G)$ of a finite group G. The former is defined as the smallest genus of those pseudo-real surfaces on which G acts faithfully as a group of automorphisms, some of which might reverse orientation, while the latter is the smallest genus of those pseudo-real surfaces on which G acts faithfully as a group of automorphisms, some of which do reverse orientation (when such a surface exists). In particular, we have shown that for every $g > 1$ there is some G with $\psi(G) = \psi^*(G) = g$, so that the range of each of the functions ψ and ψ^* is complete. We also found an example of a group G for which $\psi^*(G)$ is defined but $\psi(G) < \psi^*(G)$.

4. **Annalisa Conversano (Massey, NZ)**

Title: Lie groups in a first-order setting

Abstract: Several large classes of Lie groups, such as abelian, compact, or reductive, can successfully be studied in a first-order context, because they are naturally definable (as well as their Lie algebras) in structures with nice properties. On the other hand, there are connected low-dimensional Lie groups that are intractable from a model-theoretic perspective. The dividing line between the "good" and the "bad" groups is not clear at all yet. We report on the current state of this research, and illustrate basic examples of groups of matrices of small size.

5. **Rod Downey (Victoria, NZ)**

   Title: Effective classification

   Abstract: Lots of mathematics is concerned with classifying objects, usually up to isomorphism. Logic can be your friend in understanding when classification is possible, and if so how complicated is it to classify the class of objects of concern. I will use abelian groups, graphs and orderings as canonical examples. Mainly joint work with Melnikov and Ng.

6. **Rod Gover (Auckland, NZ)**

   Title: From the bending energy to quantum gravity via a singular Yamabe problem.

   Abstract: The classical bending energy of a surface is a conformal measure of its failure to be conformally spherical. This was introduced before the beginning of the last century for use in elasticity. It still features in this role including in mathematical biology. However it is most well known as the central object in the celebrated (and recently solved) geometric analysis problem known as the Willmore conjecture — which seeks a global minimiser for this "Willmore energy", as it is also called. Recently in collaborative and other work it has been shown that this surface invariant is the lowest dimensional example in a family of analogous quantities that arise by considering a singular version of the famous Yamabe problem. A rich picture emerges in which there are some nice and easily stated open problems for global geometry. The talk will be an elementary introduction to these directions and the new problems.

   This is based on joint work with Andrew Waldron, including "Conformal hypersurface geometry via a boundary Loewner-Nirenberg-Yamabe problem" in press CAG, `arXiv:1506.02723`; and "Renormalized Volume", in CMP 2017.

7. **Tadeusz Iwaniec (Syracuse, USA)**

   Title: Thirty five years of friendship and joint work in geometric function theory.

8. **Vaughan Jones (Vanderbilt, USA)**

   Title: What didn't work

   Abstract: I will talk about some wrong turnings I have taken in my career.

9. **Tim Marshall (Massey, NZ)**

   Title: The volume and mass distribution of a regular hyperbolic simplex

   Abstract: Motivated by a problem in ball-packing, we attempt to accurately estimate the volume of a regular n-dimensional hyperbolic simplex. This in turn leads to the problem of determining the mass distribution of such a simplex. We show that asymptotically, as $n \to \infty$, this approaches (after suitable rescaling) a normal distribution.

10. **Robert McLachlan (Massey, NZ)**

    Title: A bicomplex inspired by numerical analysis

    Abstract: This as-yet unnamed object is a synthesis of three things: the standard variational bicomplex of the calculus of variations (which extends the De Rham complex of differential forms); the B-series arising in the study of Runge-Kutta methods; and an extension of B-series, called aromas, of which $f \operatorname{div} f$ is the simplest example. It allows, for example, an enumeration of the divergence-free aromas, the linearly invariant combinations of an arbitrary vector field and its derivatives that are divergence free.

11. **Alexander Melnikov (Massey, NZ)**

    Title: Can an algorithm be an isomorphism invariant?

    Abstract: Vector spaces over a fixed field are classified by their dimension. Finitely generated abelian groups are classified by the sizes of their cyclic summands. Finite simple groups have a much harder classification, but the classification is purely algebraic. It turns out that in some natural mathematical classes one could use algorithmic processes to classify objects in these classes. We will see that, among all separable Banach spaces, Lebesgue spaces can be characterised by a certain algorithmic process. Only on a separable Lebesgue space this algorithm succeeds. This classification is also provably the "best" among all; I will explain what this means. Also, we will see that a certain broad class of discrete abelian groups can be classified using an algorithmic attempt to build a counter-example. The latter answers a long-standing open question of A.I. Maltcev from the 1960s.

12. **Eamonn O'Brien (Auckland, NZ)**

    Title: Deciding conjugacy in $\mathrm{GL}(n, \mathbb{Z})$

    Abstract: We report on a new algorithm that, given two rational matrices, decides if they are conjugate in $\mathrm{GL}(n, \mathbb{Z})$ and, if so, determines a conjugating matrix. This is joint work with Bettina Eick and Tommy Hofmann.

13. **Kirsi Peltonen (Aalto, Finland)**

    Public lecture

    Title: Sensual Mathematics

    Abstract: What are the possibilities for enhancing interaction between contemporary mathematics and arts? What will dialogue in different levels of education and research mean for society? This talk looks at the recent multidisciplinary activities challenging the traditions and communication of mathematics and arts that have taken place at Aalto University in Finland. These activities have provided a new type of platform to share the beauty of mathematics. Many outcomes and byproducts of our up-to-date experiments are perfect for applications in digital technologies such as programming, CAD, 3D printing, virtual- and augmented reality. Some scenarios for the future development are presented.

14. **Alan Reid (Rice, USA)**

    Title: Distinguishing certain triangle groups by their finite quotients

    Abstract: We prove that certain arithmetic Fuchsian triangle groups are profinitely rigid in the sense that they are determined by their set of finite quotients amongst all finitely generated residually finite groups. Amongst the examples are the $(2, 3, 8)$ triangle group.

15. **Tom ter Elst (Auckland, NZ)**

    Title: The Dirichlet problem without the maximum principle

    Abstract: The maximum principle plays an important role for the solution of the Dirichlet problem. Now consider the Dirichlet problem with respect to an elliptic operator

$$A = -\sum_{k,l=1}^{d} \partial_k \, a_{kl} \, \partial_l - \sum_{k=1}^{d} \partial_k \, b_k + \sum_{k=1}^{d} c_k \, \partial_k + c_0$$

    on a sufficiently regular open set $\Omega \subset \mathbb{R}^d$, where $a_{kl}, c_k \in L_\infty(\Omega, \mathbb{R})$ and $b_k, c_0 \in L_\infty(\Omega, \mathbb{C})$. Suppose that the associated operator on $L_2(\Omega)$ with Dirichlet boundary conditions is invertible. Note that in general this operator does not satisfy the maximum principle. Nevertheless, we show that for all $\varphi \in C(\partial\Omega)$ there exists a unique $u \in C(\overline{\Omega}) \cap H^1_{\mathrm{loc}}(\Omega)$ such that $u|_{\partial\Omega} = \varphi$ and $Au = 0$.

    In the case when Ω has a Lipschitz boundary and $\varphi \in C(\overline{\Omega}) \cap H^{1/2}(\overline{\Omega})$, then we show that u coincides with the variational solution in $H^1(\Omega)$.

    This is joint work with Wolfgang Arendt.

16. **Lesley Ward (South Australia, Australia)**

    Title: BMO and quasiconformal mappings on spaces of homogeneous type

    Abstract: Martin Reimann discovered a beautiful connection between quasiconformal mappings and functions of bounded mean oscillation. Namely, the logarithm of the Jacobian determinant of a quasiconformal mapping is always in BMO. In joint work with Trang Nguyen, we have generalised Reimann's theorem from functions defined on Euclidean spaces $\mathbb{R}^n$ to those defined on spaces of homogeneous type (X, d, μ), under certain conditions. As the analogue of quasiconformality we have used η-quasisymmetry. In this talk I will describe two different proofs, one modelled on Reimann's approach and one via a suitable Calderón–Zygmund decomposition, dyadic reverse-Hölder inequalities, and the one-third trick. I will also discuss our work in progress on generalisations to (X, d, μ) of Reimann's results about a second relationship: composition with a mapping satisfying some natural preconditions preserves BMO if and only if the mapping is quasiconformal.

17. **Tsukasa Yashiro (Japan)**

    Title: On the number of triple points in surface-knot diagrams

    Abstract: A surface-knot is a closed oriented connected surface smoothly embedded in 4-space. A surface-knot diagram is an image of a surface-knot under the orthogonal projection into 3-space equipped with the crossing information. A surface-knot diagram may contain double points, isolated triple points and isolated branch points. The least number of triple points for all possible surface-knot diagrams is called the triple point number. This number is a surface-knot invariant. It is known that for each positive integer k, 2k-twist spun trefoil has the triple point number 4k and that 3-twist spun trefoil has the triple point number 6. It is not known whether or not the triple point number is even. In this talk we introduce an expression of deforming diagrams with respect to triple points. Then we discuss how the number of triple points in a diagram can be changed. This research is a collaboration with Amal Al Kharusi.